Roll No .....

312

B. E. 1st Sem. (D. Scheme) Examination

May, 2007

MATHS – I

Paper : MATH-101-D

Time : Three hours] *[Maximum Marks : 100*

Before answering the question, candidate should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions selecting at least *two* from each Part.

PART – A

1. (a) Using Taylor's, expand $\sin x$ in powers of $\left(x-\frac{\pi}{2}\right)$. Hence find the value of $\sin 91°$ correct to 4 decimal places. 10

   (b) Find all the asymptotes of the cruve.

   $$y^3 - xy^2 - x^2y + x^3 + x^2 - y^2 = 0$$ 10

2. (a) If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, prove that

   $$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u.$$ 10

(b) Expand $\frac{(x+h)(y+k)}{x+h+y+k}$ in powers of h, k up to and inclusive of second degree terms. 10

3. (a) A rectangular box, open at the top, is to have a volume of 32 cc. Find the dimensions of the box requiring least material for its construction. 10

(b) Find the equation of cricular cylinder with base as the circle $x^2+y^2+z^2=9, x-y+z=3$. 10

4. (a) Test the convergence of the series :

$$\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+\ldots\ldots\ldots$$ 10

(b) Discuss the convergence of the series :

$$\frac{1}{2}+\frac{1.3}{2.4}+\frac{1.3.5}{2.4.6}+\ldots\ldots\ldots$$ 10

PART – B

5. (a) Evaluate the integral $\int_0^a \int_{x^2/a}^{2a-x} xy\, dy\, dx$ by 10

changing the order of integration.

(b) Evaluate $\int_0^{\pi} \int_0^{a(1+\cos\theta)} r^2 \cos\theta\, dr\, d\theta$ 10

6. (a) Find the volume bounded by xy plane, the cylinder $x^2+y^2=1$ and the plane $x+y+z=3$. 10

(b) Show that :

$$\int_0^1 y^{q-1}\left(\log\frac{1}{y}\right)^{p-1} dy = \frac{\Gamma p}{q^p};\ p>0,\quad q>0. \quad 10$$

7. (a) Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point $(2,-1,2)$. 10

(b) Prove that $\nabla^2 f(r) = f''(r)+\frac{2}{r}f'(r)$. 10

8. (a) Find the work done in moving a particle in the force field $\vec{F}=3x^2\,\hat{i}+(2xz-y)\,\hat{j}+z\,\hat{k}$ along the curve defined by $x^2=4y$, $3x^3=83$ from $x=0$ to $x=2$. 10

(b) Verify divergence theorem for $\vec{F}=4xz\,\hat{i}-y^2\,\hat{j}+yz\,\hat{k}$ taken over the cube bounded by $x=0$, $x=1, y=0, y=1, z=0, z=1$. 10