B.E. 2nd Semester Examination, May – 2009

MATHEMATICS-II

Paper–Math–102-E

Time allowed : 3 hours] *[Maximum marks : 100*

Note : *Attempt* ***five*** *questions in all, selecting at least* ***one*** *question from each part.*

Part–A

1. (a) Find the non-singular matrices P and Q such that PAQ is in the normal form for A. Hence find the rank of A, where

$$A = \begin{bmatrix} 1 & -1 & -1 & 2 \\ 4 & 2 & 2 & -1 \\ 2 & 2 & 0 & -2 \end{bmatrix}$$ 10

(b) Find the sum and product of the eigen values of the matrix

$$A = \begin{bmatrix} 2 & 3 & -1 \\ -2 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$ 4

(c) Determine a, b, c so that the matrix A is orthogonal, where

$$A=\begin{bmatrix}0 & 2b & c\\ a & b & -c\\ a & -b & c\end{bmatrix}$$ 6

2. (a) Test the consistency of the system of linear equations given below, if consistent find the solution :

$$x+3y-2z=0$$
$$2x-y+4z=0$$
$$x-11y+14z=0$$ 8

(b) Find the inverse transformation of

$$y_1=x_1+x_2+2x_3$$
$$y_2=2x_1+5x_2-2x_3$$
$$y_3=x_1+7x_2-7x_3$$ 8

(c) Verify Cayley-Hamilton Theorem for the matrix

$$\begin{bmatrix}2 & -3\\ 7 & -4\end{bmatrix}$$ 4

Part–B

3. (a) Solve the differential equation given below :

$$(3x^2 y^4 + 2xy)\, dx + (2x^3 y^3 - x^2)\, dy = 0$$ 10

(b) A body is heated to 110°C and placed in air at 10°C. After 1 hour its temperature is 60°C. How much additional time is required for it to cool to 30°C ? 10

4. (a) Solve the differential equation

$$(D^3 - D)\, z = 2y + 1 + 4 \cos y + 2e^y,$$

where $D \equiv \frac{d}{dy}$ 10

(b) Solve the following differential equation by the method of variation of parameters :

$$(D^2 + 4)\, y = 4 \sec^2 2x$$ 10

5. (a) Solve the following differential equation

$$x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 5y = x^2 \sin(\log x)$$ 10

(b) Determine the charge on the capacitor at any time $t > 0$ in a circuit in series having an emf given by $E(t) = 100 \sin 60t$ V, a resistor of $2\ \Omega$, an inductor of 0.1 H and a capacitor of $\frac{1}{260}$ farads if the initial current and initial charge on the capacitor are both zero. Find the steady-state solution. 10

Part–C

6. (a) Write the following function f (t) in terms of Heaviside's unit step function and hence find the Laplace transform of f (t). 6

(b) Find the Laplace inverse transform of the following :

(i) $\cot^{-1}\left(\frac{s+a}{b}\right)$

(ii) $\log \frac{s(s+1)}{(s^2+4)}$ 6

(c) State and prove convolution theorem to find inverse Laplace Transform. 8

7. (a) Solve the following system of equations :

$$\frac{dx}{dt} - 6x + 3y = 8\,e^t ;$$

$$\frac{dy}{dt} - 2x - y = 4\,e^t;$$

$$x(0) = -1, \quad y(0) = 0,$$

by Laplace transformation. 10

(b) Form partial differential equation by eliminating the arbitrary function from

$$z = xf(ax + by) + g(ax + by)$$ 10

8. (a) Solve the partial differential equation,

$$(y^2 + z^2)\,p - xy\,q + zx = 0$$ 6

(b) Solve the following partial differential equations :

(i) $yp + xq + pq = 0$

(ii) $(p - q)(z - px - qy) = 1$ 5

(c) Solve the following P.D.E. by the method of separation of variables :

$$u_{xx} = u_y + 2u, \quad \text{with} \quad u(0, y) = 0,$$

$$\left(\frac{\partial u}{\partial x}\right)_{(0,y)} = 1 + e^{-3y}$$ 9